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Counter intuitive results in a simple model of wage negotiations*

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Summary. Short-term contracts and exogenous productivity growth are introduced in a simple wage bargaining model. The equilibrium utilities corresponding to militant union behaviour are independent of the contract length. Necessary and sufficient conditions for monotonic convergence to a unique steady state are derived. Otherwise, cyclic behaviour of wage shares is inevitable. A wage decrease can occur if strike is credible, but never when strike is not credible. In the limit, as time between bargaining rounds vanishes, this paradox survives.

Keywords and Phrases: Wage bargaining, Wage dynamics, Chaos, Strike, Cyclic behaviour.

JEL Classification Numbers: C78, J50.

1 Introduction

The strategic wage bargaining model in Fernandez and Glazer [3], Haller [4] and Haller and Holden [5] is extended to allow for multiple wage contract renegotiations and productivity growth, by making the following more realistic assumptions. First, the parties can only agree upon short-term contracts. Second, contracts are incomplete in two different ways, namely a contract specifies a nominal wage that remains constant until it expires (i.e. wages cannot be contingent upon productivity growth) and future behaviour after the expiration date cannot be included in the contract. Third, the last expired contract remains in

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place until it is replaced by a new one. The latter is in accordance with the common practice in many Western countries, where it is forbidden by law that a firm unilaterally lowers wages after the expiration of a central agreement without the workers' consent, e.g. Holden [6].

In Fernandez and Glazer [3], Haller [4] and Haller and Holden [5] it is shown that wage increases only occur in case the union's threat of going on strike is credible. Furthermore, in Fernandez and Glazer [3] a brief remark is made with respect to short-term contracts: The union's minimum and maximum equilibrium[1] utility is not affected by assuming short-term contracts. The strategies that support this maximum equilibrium utility mimic everlasting contracts by having immediate agreement upon a wage increase equal to the wage increase under everlasting contracts in the first short-term contract and, after the expiration of this first contract, all future negotiations are immediately concluded with a short-term contract that features *no* wage increase. Our interpretation of these 'just one wage increase ever' strategies is as follows: The union is very militant in the first contract's negotiations by exploiting the threat of strike, while it behaves very peacefully in all future contracts' negotiations by refraining from this threat.

Since some unions are notorious for their aggressive behaviour it is more interesting to adapt the maximum equilibrium wage strategies such that the union uses the threat of strike whenever this threat is credible and only refrains from this threat in case it is not credible. The aim of our analysis is to investigate this particular 'militant union' equilibrium (MUE).[2] By doing so, the credibility issue is put into a dynamic context. Since both parties take into consideration how the current contract will affect future contracts' negotiations more interesting wage dynamics result than in Fernandez and Glazer [3].

Our model is formulated in terms of wage shares with the understanding that a new wage is negotiated. The union's MUE utility does not depend upon the length of the short-term contract, meaning that it is without loss of generality to assume everlasting contracts or, equivalently, 'just one wage increase ever' strategies, in deriving this utility. The 'just one wage increase ever' equilibrium and the MUE impose different paths of wage shares over time. Rational parties are indifferent between these two paths of wages. Psychologically, however, the MUE path may be enjoyed because wages keep up with profits, which might be considered 'fair'.

The union's threat of strike is only credible if the wage share is smaller than a certain threshold and the union's MUE utility depends upon whether or not strike is credible. The MUE dynamics for the wage share are derived from the union's MUE utility. For a large class of parameter values there is monotonic convergence towards to a unique steady state wage share. Otherwise, there is no steady state and cyclical behaviour is inevitable.

[1] By equilibrium we mean subgame perfect equilibrium.

[2] As in Fernandez and Glazer [3] and Haller and Holden [5] it is possible to derive inefficient equilibria which feature strike. Since doing so is by now a routine exercise, it is omitted.

There is a subclass of parameter values for which a voluntary decrease in the wage share (and also the wage) occurs. Such a decrease never occurs if strike fails as a credible weapon in the current and next contract's negotiations, but can occur either if strike is currently credible and the wage share is sufficiently high, or if strike is not currently credible but will be credible at the next contract's negotiations. Since credibility of strike is associated with a wage increase this result is counter intuitive. Wage decreases can only last for a short number of wage contracts and after that wages are forever increasing. Thus, a decrease in the wage share redistributes wages (and profits) over time in such a way that the long-run MUE wages must overtake the long-run wages corresponding to a path consisting of holdout until MUE wage increases can be negotiated.

Finally, following Binmore, Rubinstein and Wolinsky [1], we let the time between proposals vanish. In this limit strike is always credible for the union independent of the wage share and there is monotonic convergence to a unique steady state wage share. The union is unable to grasp the entire surplus. In this limit, the counter intuitive result of a decrease in the wage share survives.

This paper is organized as follows. In Section 3 we derive the union's MUE utility. The MUE wage dynamics and conditions for a wage decrease are derived in Section 4. In Section 5 the long run evolution of wage shares is analyzed. The limit behaviour of the MUE as the time between bargaining rounds vanishes is investigated in Section 6. Section 7 concludes the paper, while the next section introduces the model.

2 The model

Our wage bargaining model is basically the wage bargaining model in Fernandez and Glazer [3], Haller [4] and Haller and Holden [5] in which the assumption of an everlasting contract is dropped and exogenous productivity growth is included. Time is discrete and time $t \in \mathbb{N}$. In order to establish notation we define T, $T \in \mathbb{N} \setminus \{0\}$, as the contract length and $\pi - 1$, $\pi > 1$, as the growth rate of productivity corresponding to a learning-by-doing technology. The initial revenue generated by the firm is normalized to 1, meaning π^t is the revenue at time t. A holdout[3] is assumed to be Pareto efficient,[4] with per period payoff w_t for the union at time t (w_t is the wage specified by the last contract's wage which is either still valid or expired at time t) and the firm's profit at time t is equal to $\pi^t - w_t$. Each party's payoff at time t corresponding to strike is normalized to 0. The union is only allowed to strike at time t if at time t the last agreed upon contract has expired. If one of the parties receives the infinite sequence of payoffs $\langle x_t \rangle_{t=0}^{\infty}$, where x_t is the payoff at time t, then the *normalized* payoff to this party is simply $(1 - \delta) \sum_{t=0}^{\infty} \delta^t x_t$, $\delta \in (0, 1)$, where δ denotes the common discount factor, or, alternatively, the probability for a next period. We assume

[3] A holdout means that workers engage in production under the terms of the last expired contract.

[4] A discussion of inefficient holdouts is postponed to the concluding remarks.

that $\pi < \delta^{-1}$ in order to ensure that the surplus $(1-\delta)\sum_{\tau=t}^{\infty}\delta^{\tau-t}\pi^{\tau} = \frac{1-\delta}{1-\delta\pi}\cdot\pi^{t}$ is well defined.

The bargaining process is identical to Fernandez and Glazer [3]. At bargaining round t, t even, the union demands a wage and at round t, t odd, the firm offers a wage. In case a proposed wage is rejected, the union can either strike for one round or holdout. An agreed upon (wage) contract w lasts T rounds, after which bargaining starts over again. There are no negotiations or strikes during the validation time of a contract. Note that w influences future negotiations by specifying the disagreement payoff in these negotiations. It is necessary to specify which party restarts the negotiations after the expiration of each contract. In order to make the calculations less tedious we impose that T is even, which means that if a party proposes a wage that is accepted in round t, then this party is the proposing party at the restart of the negotiations at $t+T$.

The following reinterpretation of our model shows the resemblances and differences with the model of Fernandez and Glazer [3]. The settlement wage w_t at round t determines the wage share w_t/π^t at this period and vice versa. Thus, it can be said that at round t the parties negotiate for the wage share, denoted as y_t, with the understanding that they negotiate the wage. If the parties agree upon the wage share y at round t, then the wage will be equal to $y\cdot\pi^t$ for the next T rounds starting at t and meanwhile the wage share $y_{t+\tau}=\pi^{-\tau}y$, $\tau=0,\dots,T-1$, declines over time due to productivity growth. So, our model introduces declining wage shares over time into the framework of Fernandez and Glazer [3] combined with short-term contracts. Alternatively, nominal wages and inflation lead to a declining real wage over time, e.g. Holden [6]. In order to facilitate a comparison with the results in the literature, our analysis will be in terms of the wage share.

Note that $w_t \in \left(0, \frac{1-\delta}{1-\delta\pi}\cdot\pi^t\right]$ translates into $y_t \in \left(0, \frac{1-\delta}{1-\delta\pi}\right]$, where $\frac{1-\delta}{1-\delta\pi} > 1$. Thus, it is implicitly assumed that the firm has access to a perfect capital market in case $y_t > 1$. This assumption is without loss of generality, because the main results in this paper can also be derived if $y_t \in (0,1]$.

This reinterpretation also implies that the firm is more patient than the union. During the terms of an agreement the entire productivity growth accrues to the firm, because the wage to be paid remains fixed. So, effectively the firm has $\delta\pi > \delta$ as its discount factor. For the union the wage share during an agreement declines at the rate of π^{-1} per round, partially offsetting the discount factor $\delta\pi$. So, the union still values the future with discount factor equal to δ and is therefore less patient than the firm. Fortunately, the critique in Bolt [2] does not apply.

Finally, the declining wage share is associated with $\pi > 1$. For $\pi = 1$ the wage share does not decline. So, the fundamentals of the model under $\pi > 1$ change at the border case $\pi = 1$, i.e. there is a discontinuity at $\pi = 1$ in the dynamics of the model. The results for $\pi = 1$ are briefly discussed in the relevant sections.

3 The militant union equilibrium utilities

In this section we derive the union's MUE utility. As in Fernandez and Glazer [3], Haller [4] and Haller and Holden [5] the union's minimum equilibrium *utility* is $y_t \cdot \pi^t = w_t$, because the union can secure this payoff simply by holding out forever and never proposing nor agreeing upon wage shares that prescribe a share below y_t. Furthermore, similar as in the references above, the union's maximum equilibrium *utility* is attained if the union adopts the threat of a stutter strike, i.e. strike if the firm declines a proposal and hold out if the union declines an offer. However, there is a threshold θ, $\theta > 0$,[5] and the MUE strategies prescribe stutter strike iff $y_t \leq \theta$. Otherwise, i.e. $y_t > \theta$, the union resorts to holdout. The level of θ we are interested in is the maximum level at which stutter strike is credible. So, the militant union threatens to go on stutter strike whenever it is credible and it resorts to holdout otherwise. If the union does not carry out the threat of a stutter strike in case $y_t \leq \theta$, then an immediate switch to the strategies corresponding to the union's minimum equilibrium utility occurs (no punishment is necessary for $y_t > \theta$). The latter is only needed in the derivation of θ and will be neglected in the rest of the analysis.

First, let the function $V_i^j(y_t, t)$, $i, j = 1, 2$, denote party i's equilibrium continuation payoff at the *start* of round t and from this round onward when party j is the proposing party as function of y_t. The militant union strategies feature immediate agreement at the expiration date of an old contract and, therefore, this equilibrium is Pareto efficient. Since $V_U^j(y_t, t)$ is the normalized discounted value of an infinite stream of wages and $V_F^j(y_t, t)$ is the firm's value for the stream of revenues minus the discounted stream of wages (with the same discount factor) we have that $V_U^j(y_t, t) + V_F^j(y_t, t) = \frac{1-\delta}{1-\delta\pi}\pi^t$, $j = U, F$. Thus, the firm's value functions $V_F^j(y_t, t) = \frac{1-\delta}{1-\delta\pi}\pi^t - V_U^j(y_t, t)$ can be ignored.

Second, if the wage share at t, t even, exceeds the threshold θ, then stutter strike is not credible. However, due to the productivity growth, going on strike will become credible at $t + \ell^*(y_t)$, where $\ell^*(y_t)$ is the smallest *even* integer that solves

$$\ell^*(y_t) = \arg\min_{\ell/2 \in \mathbb{N}} \ell, \text{s.t.} \quad y_t \cdot \pi^{-\ell} \leq \theta.$$

Note that $\ell^*(y_t) = 0$ in case $y_t \leq \theta$ at t even. For t is odd, strike becomes credible at $t + \ell^*\left(y_t \cdot \pi^{-1}\right) + 1$.

The following proposition characterizes the union's MUE utility and the threshold θ. All proofs are deferred to the appendix.

Proposition 1 *The value function $V_U^U(y_t, t)$, t even, is given by*

$$V_U^U(y_t, t) = \begin{cases} \left[\frac{\delta}{1+\delta} y_t + \frac{1-\delta}{1-\delta^2\pi^2}\right]\pi^t, & y_t \leq \frac{\delta^2\pi^2(1-\delta^2)}{1-\delta^2\pi^2}, \\ \left[\frac{1+\delta-\delta^{\ell^*(y_t)}}{1+\delta} y_t + \frac{(1-\delta)(\delta\pi)^{\ell^*(y_t)}}{1-\delta^2\pi^2}\right]\pi^t, & y_t > \frac{\delta^2\pi^2(1-\delta^2)}{1-\delta^2\pi^2}. \end{cases}$$

Moreover, $V_U^U(y_t, t) \geq y_t \cdot \pi^t$ for all $y_t \in \left[0, \frac{1-\delta}{1-\delta\pi}\right]$.

[5] Actually, θ is the level for all t that are even, while the level for t odd is simply $\pi\theta$.

The threshold $\theta = \frac{\delta^2\pi^2(1-\delta^2)}{1-\delta^2\pi^2} \in \left(\delta^2\frac{1-\delta}{1-\delta\pi}, \frac{1-\delta}{1-\delta\pi}\right)$ is increasing in π, $\pi \in \left(1, \delta^{-1}\right)$. Thus, productivity growth relaxes the credibility constraint. Furthermore, the value function is independent of the contract length and, therefore, also holds for everlasting contracts, i.e. $T = \infty$. Thus, the value function also represents the union's utility associated with the maximum wage equilibrium strategies in Fernandez and Glazer [3] if productivity growth would be incorporated. Consequently, the latter strategies and the MUE strategies only differ with respect to the intertemporal distribution of per-round utilities.

The value function is discontinuous and non-monotonic at $y_t = \pi^{2k}\theta$, $k \in \mathbb{N}$, because $\ell^*\left(\pi^{2k}\theta\right) = 2k < 2k + 2 = \ell^*(y_t)$ for $y_t \in \left(\pi^{2k}\theta, \pi^{2(k+1)}\theta\right]$. The coefficient $\frac{1+\delta-\delta^{2k}}{1+\delta}$ of the linear term jumps upwards if k increases, while the constant term $\frac{(1-\delta)(\delta\pi)^{2k}}{1-\delta^2\pi^2}$ jumps downward if k increases. Thus, slightly increasing y_t at $\pi^{2k}\theta$ means that the value function jumps downward and that the slope of this linear function becomes steeper. Furthermore, as π decreases the number of points where the value function is discontinuous increases, because each interval $\left(\pi^{2k}\theta, \pi^{2(k+1)}\theta\right]$ contracts.

The results for the case $\pi = 1$ can be derived similarly as in the proof of Propositions 1 after substitution of $\pi = 1$ and a minor modification due to $\ell^*(y_t) = \infty$ for $y_t > \theta$. Then the same expressions as in Fernandez and Glazer [3] result, i.e.

$$V_U^U(y_t, t) = \begin{cases} \frac{\delta}{1+\delta}y_t + \frac{1}{1+\delta}, & y_t \leq \delta^2, \\ y_t, & y_t > \delta^2, \end{cases}$$

which is only discontinuous at $y_t = \theta = \delta^2$. Furthermore, if $\pi = 1$, then the union's utility is stuck at y_t if $y_t > \theta$, whereas $\pi > 1$ implies that the union's utility increases even if $y_t > \theta$. So, the discontinuity in the dynamics of the model is reflected in a discontinuity in the results for $\pi = 1$ and $\pi > 1$.

4 The militant union equilibrium proposals

In this section the dynamics of the wage share under MUE strategies are derived. Since T is even the MUE strategies induce an infinite sequence of consecutive contracts that are all proposed by the union and, therefore, we restrict attention to the wage share proposed by the union. Basically, the wage share y proposed by the union is the solution of

$$\left(1 - \delta^T\right) y \cdot \pi^t + \delta^T V_U^U\left(y \cdot \pi^{-T}, t + T\right) = V_U^U(y_t, t), \tag{1}$$

where $y \cdot \pi^{-T}$ is the wage share at the expiration date of the contract. The value function of Proposition 1 is not monotonic and, therefore, not invertible. This means that (1) can admit more than one solution y in case the solution for y is larger than $\pi^T\theta$. However, this is not automatically true for all $y > \pi^T\theta$. Furthermore, the discontinuities in the value function would carry over to the MUE proposal. In order to avoid these complications, we approximate $\ell^*(y_t)$ with $(\ln\pi)^{-1}\ln\frac{y_t}{\theta}$ if $y_t > \theta$. Thus, for $y_t > \theta$,

$$
\begin{aligned}
V_U^U(y_t,t) &= \left[\frac{1+\delta-\delta^{(\ln\pi)^{-1}\ln\frac{y_t}{\theta}}}{1+\delta}y_t + \frac{(1-\delta)(\delta\pi)^{(\ln\pi)^{-1}\ln\frac{y_t}{\theta}}}{1-\delta^2\pi^2}\right]\pi^t \\
&= \left[y_t + (1-\delta)\left(\frac{y_t}{\theta}\right)^{1+\frac{\ln\delta}{\ln\pi}}\right]\pi^t,
\end{aligned}
$$

where the latter expression follows after making use of $a^\ell = e^{\ell\ln a} = \left(\frac{y_t}{\theta}\right)^{\ln a/\ln\pi}$. The approximated value function is continuous in y_t and coincides with the true value function if $y_t = \pi^{2k}\theta$, $k \in \mathbb{N}$. The countable number of points on the interval $\left(\theta, \frac{1-\delta}{1-\delta\pi}\right]$ for which the two functions coincide goes to infinity as π goes to 1, which is the case in Section 6. So, the continuous value function approximates the true value function for π close enough to 1. The approximation overestimates the true value function, because implicitly it is as if the union proposes at real time $t + (\ln\pi)^{-1}\ln\frac{y_t}{\theta} \le t + \ell^*(y_t)$, t even. This bias has two opposite effects: If the approximation is used in the right hand side of (1), i.e. $y_t > \theta$, then y adjusts upwards. Whereas, if the approximation is used in the left hand side, i.e. $y > \pi^T\theta$, then y adjusts downwards.

The next proposition only states the wage share proposed by the union in case this party did not deviate in the past, because otherwise the union's minimum equilibrium utility strategies prescribe $y(y_t) = y_t$.

Proposition 2 *At round t, t even, the union proposes $y(y_t)$ given by*

$$
\begin{cases}
\frac{\delta}{1+\delta-\delta^T}y_t + \frac{\left(1-\delta^T\pi^T\right)\left(1-\delta^2\right)}{\left(1+\delta-\delta^T\right)\left(1-\delta^2\pi^2\right)}, & \text{if } y_t \le \min\{\theta^*,\theta\}, \\
y^*, & \text{if } y_t \in (\theta^*,\theta], \\
\widetilde{y}, & \text{if } y_t \in (\theta, R_1) \\
\frac{1+\delta}{1+\delta-\delta^T}y_t + \frac{1+\delta}{1+\delta-\delta^T}\left((1-\delta)\left(\frac{y_t}{\theta}\right)^{1+\frac{\ln\delta}{\ln\pi}} - \frac{\delta^T\pi^T(1-\delta)}{1-\delta^2\pi^2}\right), & \text{if } y_t \in [R_1,R_2] \\
 & \quad \cap \left(\theta, \frac{1-\delta}{1-\delta\pi}\right], \\
y_t, & \text{if } y_t \in \left(R_2, \frac{1-\delta}{1-\delta\pi}\right]
\end{cases}
$$

where $y^* > \pi^T\theta > y_t$ is the largest of the two real roots of

$$
y + (1-\delta)\left(\frac{y}{\theta}\right)^{1+\frac{\ln\delta}{\ln\pi}} - \frac{\delta}{1+\delta}y_t - \frac{1-\delta}{1-\delta^2\pi^2} = 0, \tag{2}
$$

$\widetilde{y} > \pi^T\theta > y_t$ is the largest of the two real roots of

$$
y + (1-\delta)\left(\frac{y}{\theta}\right)^{1+\frac{\ln\delta}{\ln\pi}} - y_t - (1-\delta)\left(\frac{y_t}{\theta}\right)^{1+\frac{\ln\delta}{\ln\pi}} = 0, \tag{3}
$$

the threshold $\theta^* = \delta^{-3}\pi^{-2}\left[\left(1+\delta-\delta^T\right)\delta^2\pi^{T+2} - \left(1-\delta^T\pi^T\right)\right]\cdot\theta$ and $R_1 \le \pi^T\theta \le R_2$ are the two positive, real roots of

$$
x + (1-\delta)\left(\frac{x}{\theta}\right)^{1+\frac{\ln\delta}{\ln\pi}} - \frac{1-\delta}{1-\delta^2\pi^2}\pi^T\left(\delta^T + \delta^2\pi^2\left(1+\delta-\delta^T\right)\right) = 0.
$$

A discussion of these results is postponed. First, two mutually exclusive cases are distinguished, namely $\theta^* < \theta$ and $\theta^* \geq \theta$, for which sharper results can be obtained. The case $\theta^* \geq \theta$ reduces to the following condition

$$\delta^2\pi^2 \leq \delta^{-1}\left(\pi^{T+2}\left(\delta^2+\delta^3\right)+\delta^T\pi^T\left(1-\delta^2\pi^2\right)-1\right). \tag{4}$$

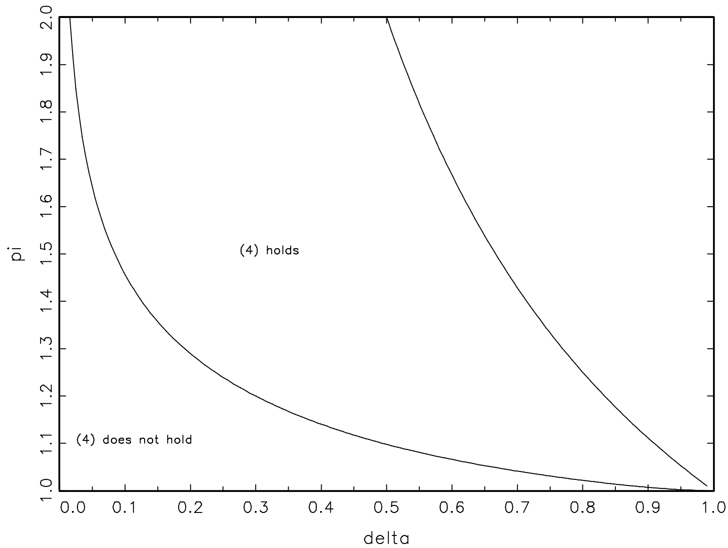

Figure 1. Condition (4) in (δ, π)-space, $T = 10$

Figure 1 illustrates condition (4). This condition is satisfied for all points on or above the lower curve and below the upper curve $\pi = \delta^{-1}$. As T increases the lower curve shifts downwards, thereby enlarging this area. Therefore, there is a nonempty area of positive measure in the (δ, π, T)-space for which condition (4) holds. The area of condition (4) is separated from the line $\pi = 1$. Similar considerations hold for the subclass excluded by condition (4).

4.1 The case condition (4) holds

The first proposition discards two of the five cases of Proposition 2.

Proposition 3 *If condition (4) is satisfied, then* $\min\{\theta^*, \theta\} = \theta$, $(\theta^*, \theta] = \emptyset$, $(\theta, R_1) = \emptyset$ *and* $[R_1, R_2] \cap (\theta, 1] = \left(\theta, \pi^T\theta\right]$ *in Proposition 2. Moreover,* $y_t \leq \theta$ *implies* $y(y_t) \cdot \pi^{-T} \leq \theta$.

The first case, $y_t \in \left(\pi^T\theta, \frac{1-\delta}{1-\delta\pi}\right]$, corresponds to $\ell(y_t) > T$. That is, the union fails strike as a credible weapon for more than T rounds. Disagreement means holdout will prevail in these rounds and, therefore, the firm can postpone an increase in the wage share at no cost, which it does, i.e., $y(y_t) = y_t$. This confirms the intuition in the literature. Strike becomes credible within $\ell(y_t) < T$

rounds in the second case, $y_t \in \left(\theta, \pi^T\theta\right]$. Then strike is currently not credible, but it is credible at the expiration date of $y(y_t)$. Thus, the militant union has one more contract to go before strike becomes credible. The union faces a dilemma, disagree and hold out until $t + \ell(y_t)$ at which strike becomes available, or negotiate a new contract now. Impatience makes the union willing to accept a wage share today in order to avoid waiting until $t + \ell(y_t)$. The third case, $y_t \in [0, \theta]$, corresponds to strike is both credible in the current and next contract's negotiations. Note that once strike is credible it remains credible in all future contracts' negotiations, i.e. $y(y_t) \cdot \pi^{-T} \leq \theta$ for $y_t \leq \theta$.

The next proposition implies that the union does not always negotiate a larger wage share.

Proposition 4 *Suppose condition (4) holds. If* $\frac{1-\delta^T\pi^T}{\delta^2\pi^2\left(1-\delta^T\right)} < 1$, *then*

$$y(y_t) < y_t \Leftrightarrow y_t \in \left(\frac{1-\delta^T\pi^T}{\delta^2\pi^2\left(1-\delta^T\right)}\theta, \pi^T\theta\right] \neq \emptyset.$$

Remark 1 The condition is necessary and sufficient for a decrease in the wage share if $y_t \in \left(\frac{1-\delta^T\pi^T}{\delta^2\pi^2\left(1-\delta^T\right)}\theta, \theta\right]$. Furthermore, in case $\frac{1-\delta^T\pi^T}{\delta^2\pi^2\left(1-\delta^T\right)} \geq 1$ then there exists some $\lambda > 1$ such that a decrease in the wage share occurs if $y_t \in \left(\lambda\theta, \pi^T\theta\right]$ even.[6]

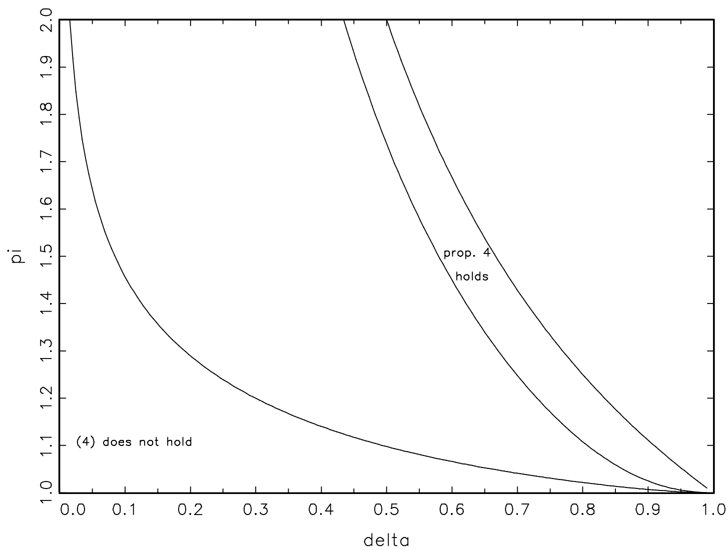

Figure 2. Proposition 4 in (δ, π)-space, $T = 10$

The condition $\frac{1-\delta^T\pi^T}{\delta^2\pi^2\left(1-\delta^T\right)} < 1$ is satisfied for all points above the middle of the three curves in Figure 2. Moreover, according to Figure 2 the area of this

[6] The bound $\lambda\theta$ corresponds to one of two roots of the polynomial in the proof of Proposition 4.

condition is contained in the area corresponding to condition (4) bounded by the lowest curve. We conjecture that this holds for any triple (δ, π, T).[7] Finally, the area in the (δ, π, T)-space for which the condition $\frac{1-\delta^T \pi^T}{\delta^2 \pi^2 (1-\delta^T)} < 1$ holds has positive measure, because substitution of $\pi = \delta^{-1}$ implies $0 < 1$ for all T.

The result in Proposition 4 differs from the results in the literature where an increase in the wage (share) is identified with strike being credible. Clearly, it is counter intuitive that a decrease in the wage share (and the wage itself) occurs in or near the region where strike is credible, but not where strike is not credible. This implies that for the union the initial wage decrease is compensated with an acceleration of the increase of the wage share in the future.

4.2 The case condition (4) does not hold

The next proposition states that all five cases of Proposition 2 occur if condition (4) does not hold.

Proposition 5 *If condition (4) does not hold, then* $\min\{\theta^*, \theta\} = \theta^*$, $(\theta^*, \theta] \neq \emptyset$, $(\theta, R_1) \neq \emptyset$ *and* $[R_1, R_2] \cap (\theta, 1] = [R_1, R_2] \neq \emptyset$ *in Proposition 2.*

The first case, $y_t \in [0, \theta^*]$, corresponds to strike is both credible in the current and next contract's negotiations. The union always negotiates a larger wage share.[8] The second case, $y_t \in (\theta^*, \theta]$, is the transition from strike is credible in the current contract's negotiations to strike is not credible at the next contract's negotiations. So, the militant union has one more contract to go before strike ceases to be credible. The negotiated wage share is larger than the current wage share, because $y(y_t) = y^* > \pi^T \theta > y_t$. The third case, $y_t \in (\theta, R_1)$, with $R_1 < \pi^T \theta$, corresponds to strike is not credible before $t + \ell(y_t)$, where $\ell(y_t) < T$, at the current contract's negotiations and it will neither be credible at the expiration date of $y(y_t) = \widetilde{y}$. Despite the fact that strike is not credible the union negotiates a substantially larger wage share $\widetilde{y} > \pi^T \theta > y_t$. The reason is that both parties anticipate the large increase of the wage share at $t + \ell(y_t)$ of the second case if they would not agree for the next $\ell(y_t) < T$ rounds and impatience makes the union willing to accept a lower increase in the wage share today in order to avoid the waiting. The fourth case, $y_t \in [R_1, R_2]$, corresponds to the situation in which strike is not credible at time t but strike is credible at the expiration date of $y(y_t)$. Thus, the militant union has one more contract to go before strike becomes credible. Conditions for a decrease in the negotiated wage share are not derived. The last case, $y_t \in \left(R_2, \frac{1-\delta}{1-\delta\pi}\right]$, with $R_2 \geq \pi^T \theta$, corresponds to $\ell(y_t) > T$, i.e. the union lacks strike as a credible weapon now and at the end of the new contract. In that case there is no increase in the wage share, because the future threat of strike lies too far away in the future.

[7] For 10.000 random triples (δ, π, T) we numerically verified whether condition (4) holds if the condition of Proposition 4 holds. For all cases this was found true.

[8] This follows directly from the proof of Proposition 6 below, where monotonic convergence to x^* is shown, and $x^* > \theta$ if condition (4) does not hold.

5 The wage share in the long run

Similar as in the previous section we distinguish two exclusive cases.

5.1 The case condition (4) holds

The dynamics on $[0, \theta]$ admit a steady state wage share and, moreover, there is monotonic convergence to this steady state. The wage share $x^* \in [0, 1]$ is a steady state wage share if $y_t = x^*$ implies that $y(y_t) \cdot \pi^{-T} = x^*$. So, at time $t+T$, i.e. the expiration date of $y(y_t)$, the wage share will be equal to the wage share at t.

Proposition 6 *If condition (4) is satisfied, then there is monotonic convergence in wage shares to*

$$x^* = \frac{\left(1 - \delta^T \pi^T\right)}{\delta^2 \pi^2 \left(\pi^T \left(1 + \delta - \delta^T\right) - \delta\right)} \theta \leq \theta$$

for all $y_t \in \left[0, \frac{1-\delta}{1-\delta\pi}\right]$. *Moreover, if* $\frac{1-\delta^T \pi^T}{\delta^2 \pi^2 \left(1-\delta^T\right)} < 1$, *then* $x^* < \frac{1-\delta^T \pi^T}{\delta^2 \pi^2 \left(1-\delta^T\right)} \theta < \theta$.

Note that the wage share (and, consequently, also the wage) is forever monotonically increasing if the initial wage share lies below x^*. However, if initially the wage share lies above x^*, then even a militant union which has strike as a credible option at its disposal cannot prevent that the wage share will decrease over time. A decrease in the wage share cannot be ruled out according to Proposition 4. The number of consecutive contracts featuring a wage decrease is bounded from above, because the long-run wage share approximates x^* and an increase to $\pi^T x^*$ is needed in order to sustain the steady state wage share. So, after a finite number of contracts, wages exceed the initial level.

5.2 The case condition (4) does not hold

If condition (4) does not hold, then the steady state x^* of Proposition 6 exceeds θ and lies outside the interval $[0, \theta^*]$. Still the wage share monotonically increases over time on the domain $[0, \theta^*]$ and, eventually, after a finite number of contracts, the wage share will be larger than θ^*.[9] It follows that we have cyclical behaviour of wage shares.

This can be seen as follows. After y_t has left the interval $[0, \theta]$, y_t enters either the interval $[R_1, R_2]$ or $(\theta, R_1) \cup \left(R_2, \frac{1-\delta}{1-\delta\pi}\right)$. In the former case, the wage share at the end of the contract is in the interval $[0, \theta]$, in the latter case it could either stay in this interval or enter the interval $[R_1, R_2]$. The most one can obtain

[9] This follows directly from the proof of Proposition 6 if $\left[0, \frac{1-\delta}{1-\delta\pi}\right]$ is taken as the domain of the dynamics and observing that monotonic convergence to x^* on $\left[0, \frac{1-\delta}{1-\delta\pi}\right]$ holds even if $x^* > \theta$.

are limit cycles (if these exist at all). This is illustrated in the next numerical example.

Take $\delta = 0.8$, $\pi = 1.02$, $T = 4$. Then $\theta^* = 0.6$, $\theta = 0.717$, $R_1 = 0.777$ and $R_2 = 0.799$. It turns out that in this case, independent of the starting value y_0, the evolution of the wage share converges to the cycle described in Table 1. Note that in this case a wage decrease occurs for $y_t \in [R_1, R_2]$.

Table 1

Current wage share	*Interval*	*Proposal*
0.778	$[R_1, R_2]$	0.776
0.717	(θ^*, θ)	0.897
0.829	$(R_2, \frac{1-\delta}{1-\delta\pi})$	0.829
0.766	(θ, R_1)	0.812
0.750	(θ, R_1)	0.834
0.770	(θ, R_1)	0.807
0.745	(θ, R_1)	0.842
0.778	$[R_1, R_2]$	0.776

Finally, the line $\pi = 1$ is one of the boundaries of the area where condition (4) does not hold. For $\pi = 1$ we obtain $(\theta, R_1) = \emptyset$ and $[R_1, R_2] = \emptyset$ in Proposition 2.[10] Thus,

$$y(y_t) = \begin{cases} \frac{\delta}{1+\delta-\delta^T} y_t + \frac{1-\delta^T}{1+\delta-\delta^T}, & \text{if } y_t \in \left[0, \delta^2 - \frac{1-\delta^2}{\delta}\right], \\ \frac{\delta}{1+\delta} y_t + \frac{1}{1+\delta}, & \text{if } y_t \in \left(\delta^2 - \frac{1-\delta^2}{\delta}, \delta^2\right], \\ y_t, & \text{if } y_t \in \left(\delta^2, 1\right], \end{cases}$$

for sufficiently large $\delta < 1$. We mention that *i)* only a finite number of wage increases can take place and *ii)* the last contract featuring a wage increase specifies a wage in between δ^2 and $1 - \delta(1-\delta) < 1$. Hence, $\pi = 1$ and MUE strategies impose a wage ceiling and once strike ceases to be credible it remains not credible forever. So, for $\pi = 1$ there is a wage ceiling above $\theta = \delta^2$ while cyclic behaviour results for $\pi > 1$. So, the discontinuity in the model at $\pi = 1$ translates to a discontinuity in results.

6 Limit results

In this section we follow the literature on strategic bargaining by letting the time between bargaining rounds vanish (e.g. Binmore et al. [1]) and, meanwhile, maintaining a constant level of the contract length measured in real time.

In order to make the analysis precise we define Δ, $\Delta > 0$, as the time between two bargaining rounds, $\delta = e^{-r\Delta}$, where r denotes the interest rate,

[10] The following statements can be derived similarly as in the *proof* of Proposition 2 after substitution of $\pi = 1$ and a minor modification due to $\ell(y_t) = \infty$ in case $y_t > \delta^2$. Furthermore, $[R_1, R_2]$ is empty, because once $y_t > \theta$ strike remains not credible forever.

and $\pi = e^{\rho\Delta}$, where ρ denotes the growth rate of productivity. The assumption $1 < \pi < \delta^{-1}$ implies that $0 < \rho < r$. The contract length $L \in \mathbb{R}_{++}$ is measured in real time. Since $L = T\Delta$ and L is kept constant T has to adjust if Δ goes to 0, i.e. $T = L/\Delta$.[11] For simplicity we neglect that $T = L/\Delta$ should be an even integer. The following theorem states the limit results.

Theorem 1 *In the limit, as Δ goes to 0, strike is credible for every $y_t \in \left[0, \frac{r}{r-\rho}\right]$, the union proposes the wage share $\frac{1}{2-e^{\rho L}}\left[y_t + \left(1 - e^{-(r-\rho)L}\right)\frac{r}{r-\rho}\right]$ and its MUE utility is $\frac{1}{2}\left[y_t + \frac{r}{r-\rho}\right]e^{\rho t}$. Moreover, there is monotonic convergence in wage shares to $\frac{\left(1-e^{-(r-\rho)L}\right)r}{\left(e^{\rho L}\left(2-e^{-rL}\right)-1\right)(r-\rho)} < \frac{r}{r-\rho}$ and the wage share decreases if $y_t \in \left(\frac{\left(1-e^{-(r-\rho)L}\right)r}{\left(1-e^{-rL}\right)(r-\rho)}, \frac{r}{r-\rho}\right] \neq \emptyset$.*

The threshold θ (and $\frac{1-\delta}{1-\delta\pi}$) converges to $\frac{r}{r-\rho}$ as the time between bargaining rounds vanishes. The reason is that the equilibrium condition for strike to be credible is given by $y_t \cdot e^{\rho t} \leq \left(1 - e^{-r\Delta}\right) \cdot 0 + e^{-r\Delta} V_U^F\left(y_t \cdot e^{-\rho\Delta}, t+1\right)$, t even. Since $V_U^F\left(y_t \cdot e^{-\rho\Delta}, t+1\right) > y_t \cdot e^{\rho t}$ for all $\Delta > 0$ this condition requires $e^{-r\Delta}$ sufficiently close to 1, which holds for sufficiently small $\Delta > 0$. This equilibrium condition easily generalizes to a large class of extended models, provided $y_t \cdot e^{\rho t}$ is replaced by the union's minimum equilibrium utility in such model. Furthermore, the union is unable to grasp the entire surplus $\frac{r}{r-\rho}e^{\rho t}$. A decrease in the wage share will occur for sufficiently large wage shares.

Finally, the union's limit MUE utility corresponds to the Nash bargaining solution with *everlasting* contracts and disagreement point $\left(y_t \cdot e^{\rho t}, 0\right)$, because

$$\max_{w} \left(w - y_t \cdot e^{\rho t}\right)\left(\frac{r}{r-\rho} \cdot e^{\rho t} - w\right) \Rightarrow w = \frac{1}{2}\left(y_t + \frac{r}{r-\rho}\right)e^{\rho t}.$$

This means that the limit results can be obtained by a simple two-step procedure. First, compute the union's limit MUE utility by applying this Nash bargaining solution and, second, derive the dynamics of the wage share from this utility.

7 Concluding remarks

In this paper we have extended the wage bargaining model by Fernandez and Glazer [3], Haller [4] and Haller and Holden [5], by allowing for finite contract length and productivity growth. By doing so, credibility over time of the strike threat enters the analysis. However, the credibility issue vanishes as time between bargaining rounds goes to zero and matters simplify in the limit. The results then hint that a large class of extended models can be analyzed using

[11] Note that keeping T fixed would mean that $L = T\Delta \to 0$ as $\Delta \to 0$. Then, in the limit, parties would constantly negotiate new contracts which expire instantaneously at the moment of conclusion. Clearly, this is unrealistic.

the Nash bargaining solution with the appropriate disagreement points. For instance one could think of inefficient holdouts, e.g. Holden [6] and Moene [8], or introducing competition among strategic options for the union, such as holdout, work-to-rule and strike, e.g. Houba and Bolt [7]. Inefficient holdouts only require a minor modification of the union's minimum equilibrium utility in our analysis. Competition among strategic options would not alter the conclusions of Theorem 1, because in the limit all actions will be credible and strike is the union's most effective action among the credible actions.

Our analysis complements the study in Holden [6], where the Nash bargaining solution determines the nominal wage in a small macroeconomic model with inflation, endogenous employment, decentralized wage bargaining and one-year contracts. Furthermore, the credibility issue is not addressed by assuming that the disagreement point corresponds to inefficient holdout. Our results hint that the credibility issue vaporizes in the limit and that, alternatively, the disagreement point corresponding to stutter strike could have been taken. In Holden [6] workers also have an outside option and conditions for a binding outside option are derived. So, wage decreases are implicitly present in Holden [6]. Theorem 1 also hints that this result can still be expected in Holden [6] if a militant union would be assumed.

Appendix

Proof of Proposition 1

Suppose there is a threshold θ such that at t is even strike is credible iff $y_t \leq \theta$. If $y_t > \theta$, then strike becomes credible at $t + \ell$ "even", where ℓ denotes $\ell(y_t)$. There are two cases to be considered.

Case 1 $y_t > \theta$. Then $t + \ell$ is the first round strike will be credible again and, hence, $V_U^U(y_{t+\ell}, t + \ell)$ corresponds to the function at which strike is credible. Until $t + \ell$ strike is not credible and $y_t \cdot \pi^t$ is the union's disagreement payoff. The firm's problem at $\tau + 1$, $\tau = t + \ell - 2, t + \ell - 4, \ldots, t$ (τ even), is given by

$$\begin{aligned} & \frac{1-\delta}{1-\delta\pi}\pi^{\tau+1} - V_U^F(y_{\tau+1}, \tau+1) \\ = \quad & \max_{y_F} \frac{1-\delta}{1-\delta\pi}\pi^{\tau+1} - \left(1-\delta^T\right) y_F \cdot \pi^{\tau+1} - \delta^T V_U^F\left(y_F \cdot \pi^{-T}, \tau + T + 1\right), \end{aligned}$$

s.t.

$$\begin{aligned} & \left(1-\delta^T\right) y_F \cdot \pi^{\tau+1} + \delta^T V_U^F\left(y_F \cdot \pi^{-T}, \tau+T+1\right) \\ = \quad & (1-\delta) y_{\tau+1} \cdot \pi^{\tau+1} + \delta V_U^U\left(y_{\tau+1} \cdot \pi^{-1}, \tau+2\right). \end{aligned}$$

Substitution of the constraint into the objective function and rewriting yields

$$V_U^F(y_{\tau+1}, \tau+1) = (1-\delta) y_{\tau+1} \cdot \pi^{\tau+1} + \delta V_U^U\left(y_{\tau+1} \cdot \pi^{-1}, \tau+2\right). \qquad (5)$$

Similarly, the union's problem at τ given by

$$V_U^U(y_\tau,\tau)=\max_{y_U}\left(1-\delta^T\right)y_U\cdot\pi^\tau+\delta^T V_U^U\left(y_U\cdot\pi^{-T},\tau+T\right),$$

s.t.

$$\begin{aligned}&\frac{1-\delta}{1-\delta\pi}\pi^\tau-\left(1-\delta^T\right)y_U\cdot\pi^\tau-\delta^T V_U^U\left(y_U\cdot\pi^{-T},\tau+T\right)\\ =\ &\frac{1-\delta}{1-\delta\pi}\pi^\tau-(1-\delta)y_\tau\cdot\pi^\tau-\delta V_U^F\left(y_\tau\cdot\pi^{-1},\tau+1\right)\end{aligned}$$

yields

$$V_U^U(y_\tau,\tau)=(1-\delta)y_\tau\cdot\pi^\tau+\delta V_U^F\left(y_\tau\cdot\pi^{-1},\tau+1\right). \tag{6}$$

Furthermore, if the parties would not agree at τ then at $\tau+1$ we have that $y_{\tau+1}=y_\tau\cdot\pi^{-1}$. Making use of this relation and substitution of (5) into (6) yields the recursive relation

$$V_U^U(y_\tau,\tau)=\left(1-\delta^2\right)y_\tau\cdot\pi^\tau+\delta^2 V_U^U\left(y_\tau\cdot\pi^{-2},\tau+2\right),\tau=t+\ell-2,t+\ell-4,\ldots,t.$$

Solving the recursion yields

$$V_U^U(y_t,t)=\left(1-\delta^\ell\right)y_t\cdot\pi^t+\delta^\ell V_U^U\left(y_t\cdot\pi^{-\ell},t+\ell\right),\text{for } y_t>\theta \tag{7}$$

and $V_U^U\left(y_t\cdot\pi^{-\ell},t+\ell\right)$ refers to case 2, i.e. $y_t\cdot\pi^{-\ell}\leq\theta$.

Case 2 $y_t\leq\theta$ at t even. Then going on strike is credible. The union's problem (6) at t, t is even, is different and is now given by

$$V_U^U(y_t,t)=\max_{y_U}\left(1-\delta^T\right)y_U\cdot\pi^t+\delta^T V_U^U\left(y_U\cdot\pi^{-T},t+T\right)$$

s.t.

$$\begin{aligned}&\frac{1-\delta}{1-\delta\pi}\pi^t-\left(1-\delta^T\right)w_U-\delta^T V_U^U\left(w_U,\pi^T F_t\right)\\ &=\delta\left[\frac{1-\delta}{1-\delta\pi}\pi^{t+1}-V_U^F\left(y_t\cdot\pi^{-1},t+1\right)\right],\end{aligned}$$

$$V_U^U\left(y_U\cdot\pi^{-T},t+T\right)=\left(1-\delta^\ell\right)w_U+\delta^\ell V_U^U\left(y_U\cdot\pi^{-T-\ell},t+T+\ell\right).$$

where ℓ denotes $\ell\left(y_U\cdot\pi^{-T}\right)$. The second constraint comes from case 1 and is necessary in order to take into account y_U's such that $y_U\cdot\pi^{-T}>\theta$ at $t+T$. Note that $\ell\left(y_U\cdot\pi^{-T}\right)=0$ for $y_U\cdot\pi^{-T}\leq\theta$ and then this constraint is superfluous. As before, substitution of the first constraint into the objective function yields

$$V_U^U(y_t,t)=(1-\delta\pi)\frac{1-\delta}{1-\delta\pi}\pi^t+\delta V_U^F\left(y_t\cdot\pi^{-1},t+1\right). \tag{8}$$

The firm's problem at $t+1$, t is even, does not change and, hence, equation (5) is also valid in this case for $\tau+1=t+1$. If the parties would not agree at t, t is even, then at $t+1$ (odd) we have that $y_{t+1}=y_t\cdot\pi^{-1}$. Making use of this relation and substitution of (5) (for $\tau+1=t+1$) into (8) yields

$$V_U^U(y_t,t)=(1-\delta)\pi^t+\delta(1-\delta)y_t\cdot\pi^t+\delta^2 V_U^U\left(y_t\cdot\pi^{-2},t+2\right),\ \text{for } y_t\leq\theta. \tag{9}$$

It is easy to verify that

$$V_U^U(y_t,t)=\left[\frac{\delta}{1+\delta}y_t+\frac{1-\delta}{1-\delta^2\pi^2}\right]\pi^t,\ \text{for } y_t\le\theta,$$

is a solution of (9). Then the expression in (7) becomes

$$V_U^U(y_t,t)=\left[\frac{1+\delta-\delta^\ell}{1+\delta}y_t+\frac{(1-\delta)\,\delta^\ell\pi^\ell}{1-\delta^2\pi^2}\right]\pi^t,\ \text{for } y_t>\theta. \tag{10}$$

It is easy to verify that $V_U^U(y_t,t)\ge y_t\cdot\pi^t$ for all y_t.

Finally, strike at t, t even, is credible iff $y_t\cdot\pi^t\le(1-\delta)\cdot 0+\delta V_U^F\left(y_t\cdot\pi^{-1},t+1\right)$. At t is odd, we have

$$V_U^F(y_t,t)=(1-\delta)+y_t\cdot\pi^t+\delta V_U^U\left(y_t\cdot\pi^{-1},t+1\right)=\frac{\delta}{1+\delta}y_t+\frac{(1-\delta)\,\delta\pi}{1-\delta^2\pi^2},$$

for $y_t\le\pi\theta$. Solving for θ in $\theta=\delta V_U^F\left(\theta\cdot\pi^{-1},t+1\right)$ yields $\theta=\delta^2\pi^2\frac{1-\delta^2}{1-\delta^2\pi^2}$ as the threshold postulated at the beginning of the proof. QED

Proof of Proposition 2. The two expressions for V_U^U on both sides of (1) imply four different cases.

First, if $y_t\le\theta$ and $y\cdot\pi^{-T}\le\theta$, then y solves (1) iff

$$\left(1-\delta^T\right)y+\delta^T\left(\frac{\delta}{1+\delta}y\cdot\pi^{-T}+\frac{1-\delta}{1-\delta^2\pi^2}\right)\pi^T=\frac{\delta}{1+\delta}y_t+\frac{1-\delta}{1-\delta^2\pi^2},$$

which yields the solution

$$y=\frac{\delta}{1+\delta-\delta^T}y_t+\frac{\left(1-\delta^T\pi^T\right)\left(1-\delta^2\right)}{\left(1+\delta-\delta^T\right)\left(1-\delta^2\pi^2\right)}.$$

The condition $y\cdot\pi^{-T}\le\theta$ can be rewritten as

$$y_t\le\delta^{-1}\frac{1-\delta^2}{1-\delta^2\pi^2}\left(\delta^T\pi^T\left(1-\delta^2\pi^2\right)+\pi^{T+2}\left(\delta^2+\delta^3\right)-1\right)\equiv\theta^*.$$

Combining the two conditions $y_t\le\theta$ and $y\cdot\pi^{-T}\le\theta$ yields $y_t\le\min\{\theta^*,\theta\}$.

Second, if $y_t>\theta$ and $y\cdot\pi^{-T}\le\theta$, then y solves (1) iff

$$\left(1-\delta^T\right)y+\delta^T\left(\frac{\delta}{1+\delta}y\cdot\pi^{-T}+\frac{1-\delta}{1-\delta^2\pi^2}\right)\pi^T=y_t+(1-\delta)\left(\frac{y_t}{\theta}\right)^{1+\frac{\ln\delta}{\ln\pi}}.$$

Solving for y yields

$$y=\frac{1+\delta}{1+\delta-\delta^T}y_t+\frac{1+\delta}{1+\delta-\delta^T}\left((1-\delta)\left(\frac{y_t}{\theta}\right)^{1+\frac{\ln\delta}{\ln\pi}}-\frac{\delta^T\pi^T(1-\delta)}{1-\delta^2\pi^2}\right).$$

The condition $y\cdot\pi^{-T}\le\theta$ can be rewritten as

$$y_t + (1-\delta)\left(\frac{y_t}{\theta}\right)^{1+\frac{\ln\delta}{\ln\pi}} - \frac{1-\delta}{1-\delta^2\pi^2}\pi^T\left(\delta^T + \delta^2\pi^2\left(1+\delta-\delta^T\right)\right) \leq 0. \tag{11}$$

The polynomial is decreasing for y_t small and increasing for y_t large, because $1+\frac{\ln\delta}{\ln\pi} < 0$ implies $\lim_{x\to 0} x^{1+\frac{\ln\delta}{\ln\pi}} = +\infty$ and $\lim_{x\to\infty} x^{1+\frac{\ln\delta}{\ln\pi}} = 0$, i.e. the graph is U-shaped. Furthermore, $y_t = \pi^T\theta$ is one real root, because $\left(\pi^T\right)^{1+\frac{\ln\delta}{\ln\pi}} = \delta^T\pi^T$. Therefore, there exist two positive, real roots, i.e. $R_1 \leq \pi^T\theta \leq R_2$. Combining $y_t > \theta$ and $y\cdot\pi^{-T} \leq \theta$ yields $y_t \in [R_1, R_2] \cap \left(\theta, \frac{1-\delta}{1-\delta\pi}\right]$.

Third, if $y_t > \theta$ and $y\cdot\pi^{-T} > \theta$, then y solves (1) iff

$$\left(1-\delta^T\right)y + \delta^T\left(y\cdot\pi^{-T} + (1-\delta)\left(\frac{y\cdot\pi^{-T}}{\theta}\right)^{1+\frac{\ln\delta}{\ln\pi}}\right)\pi^T = y_t + (1-\delta)\left(\frac{y_t}{\theta}\right)^{1+\frac{\ln\delta}{\ln\pi}},$$

which can be rewritten as stated in (3) by making use of $\left(\pi^{-T}\right)^{1+\frac{\ln\delta}{\ln\pi}} = \delta^{-T}\pi^{-T}$. By the same arguments as applied to (11) in the second case, (3) is U-shaped in y and there exist two positive, real roots $S_1 < S_2$, because $y = y_t$ is one of them.

Next, we prove that $S_1 < \pi^T\theta < S_2$, i.e. $S_2\cdot\pi^{-T} > \theta$, for all $y_t \in (\theta, R_1) \cup \left(R_2, \frac{1-\delta}{1-\delta\pi}\right]$, where the latter set results from $y_t > \theta$ and discarding $[R_1, R_2]$ of the second case. The arguments are as follows. Consider (3) at $y = \pi^T\theta$ as a function of $y_t \in (\theta, R_1) \cup \left(R_2, \frac{1-\delta}{1-\delta\pi}\right]$. Substitution of $y = \pi^T\theta$ into (3) yields

$$\pi^T\theta + (\delta\pi)^T(1-\delta) - y_t - (1-\delta)\left(\frac{y_t}{\theta}\right)^{1+\frac{\ln\delta}{\ln\pi}},$$

which can be rewritten as

$$\pi^T\frac{(1-\delta)}{1-\delta^2\pi^2}\left(\delta^2\pi^2\left(1+\delta-\delta^T\right)+\delta^T\right) - y_t - (1-\delta)\left(\frac{y_t}{\theta}\right)^{1+\frac{\ln\delta}{\ln\pi}}, \tag{12}$$

because

$$\begin{aligned}
\pi^T\theta + \delta^T\pi^T(1-\delta) &= \pi^T\left(\frac{\left(1-\delta^2\right)\delta^2\pi^2}{1-\delta^2\pi^2} + \frac{\left(1-\delta^2\pi^2\right)(1-\delta)\delta^T}{1-\delta^2\pi^2}\right) \\
&= \pi^T\frac{(1-\delta)}{1-\delta^2\pi^2}\left((1+\delta)\delta^2\pi^2 + \delta^T\left(1-\delta^2\pi^2\right)\right) \\
&= \pi^T\frac{(1-\delta)}{1-\delta^2\pi^2}\left(\delta^2\pi^2\left(1+\delta-\delta^T\right)+\delta^T\right).
\end{aligned}$$

But (12) is equal to minus expression (11). Since (11) is positive for $y_t \in (\theta, R_1) \cup (R_2, \frac{1-\delta}{1-\delta\pi}]$, as shown in the second case, we have that (12) is negative for such y_t. Hence, $S_1 < \pi^T\theta < S_2$ and only S_2 satisfies $y\cdot\pi^{-T} > \theta$ for all $y_t \in (\theta, R_1) \cup (R_2, \frac{1-\delta}{1-\delta\pi}]$. Finally, $y = y_t$ is one root of (3). If $\theta < y_t < R_1 \leq \pi^T\theta$, then necessarily $S_1 = y_t < \pi^T\theta$. Otherwise, $S_2 = y_t > \pi^T\theta$ for $y_t > \pi^T\theta$.
Fourth, $y < \theta$ and $y\cdot\pi^{-T} > \theta$. In that case y solves (1) iff

$$y + \delta^T \pi^T (1-\delta) \left(\frac{y \cdot \pi^{-T}}{\theta}\right)^{1+\frac{\ln \delta}{\ln \pi}} = \frac{\delta}{1+\delta} y_t + \frac{1-\delta}{1-\delta^2\pi^2},$$

which yields (2). Similarly as in the third case, (2) is U-shaped in y, and has two positive, real roots. The smallest root is not feasible, i.e. $y < \pi^T\theta$, iff the LHS of (2) is negative at $y = \pi^T\theta$ (because in that case $\pi^T\theta$ lies in between the two roots). Substitution of $y = \pi^T\theta$ yields

$$\pi^T\theta + (\delta\pi)^T (1-\delta) - \frac{\delta}{1+\delta} y_t - \frac{1-\delta}{1-\delta^2\pi^2}.$$

The latter expression is equal to 0 in $y_t = \theta^*$ (see third case) and decreases in y_t. Thus, this expression is negative iff $y_t > \theta^*$. The latter is true iff $y_t > \theta^*$. Thus, the unique feasible solution is the largest root of (2) for all $y_t \in (\theta^*, \theta)$. QED

Proof of Proposition 3. First, $\min\{\theta^*, \theta\} = \theta$ iff condition (4) holds. Next, (11) holds for $y_t = \theta$, because $\left(1-\pi^T\right)\delta^2\pi^2\frac{1+\delta}{1+\delta\pi} + (1-\delta\pi)\left(1-\delta^T\pi^T\right) \le 0$ is equivalent to condition (4). Hence, $R_1 \le \theta$ and $R_2 = \pi^T\theta$ in the proof of Proposition 2. Thus, $\left(\theta, \frac{1-\delta}{1-\delta\pi}\right] \cap [R_1, R_2] = \left(\theta, \pi^T\theta\right]$. Finally, $y(y_t) \le y(\theta) \le \pi^T\theta$ iff condition (4) holds. QED

Proof of Proposition 4. Propositions 2 and 3 imply that a wage decrease cannot occur for $y_t \in \left(\pi^T\theta, \frac{1-\delta}{1-\delta\pi}\right]$. Thus, only $y_t \le \theta$ and $y_t \in \left(\theta, \pi^T\theta\right)$ have to be investigated. First, $y_t \le \theta$. Then $y < y_t$ iff $y_t > \frac{\left(1-\delta^T\pi^T\right)}{\delta^2\pi^2\left(1-\delta^T\right)}\theta$. The interval $\left(\frac{\left(1-\delta^T\pi^T\right)}{\delta^2\pi^2\left(1-\delta^T\right)}\theta, \theta\right]$ is non-empty iff $\frac{\left(1-\delta^T\pi^T\right)}{\delta^2\pi^2\left(1-\delta^T\right)} < 1$. Second, $y_t \in \left(\theta, \pi^T\theta\right)$. Then $y < y_t$ iff

$$\delta^{T+2}\pi^2\frac{y_t}{\theta} + \left(1-\delta^2\pi^2\right)\left(\frac{y_t}{\theta}\right)^{1+\frac{\ln\delta}{\ln\pi}} - \delta^T\pi^T < 0.$$

Similar to the proof of Proposition 2 it follows that $y_t = \pi^T\theta$ is one of the two positive, real roots. This condition holds in $y_t = \theta$, i.e. the smallest root is smaller than θ, iff $\frac{1-\delta^T\pi^T}{\delta^2\pi^2\left(1-\delta^T\right)} < 1$. In that case $\pi^T\theta$ is necessarily the largest root and the polynomial is negative for all $y_t \in \left(\theta, \pi^T\theta\right]$. QED

Proof of Proposition 5. First, $\min\{\theta^*, \theta\} = \theta^* < \theta$ iff condition (4) does not hold. Next, from the proof of Proposition 3 it follows that (11) in the proof of Proposition 2 does not hold for $y_t = \theta$ if the condition (4) does not hold. Since, $\pi^T\theta$ is one of the two real, positive roots it necessarily follows that both R_1 and R_2 are larger than θ. QED

Proof of Proposition 6. Propositions 2 and 3 imply that $y_t > \theta$ can not be a steady state, because the wage share monotonically decreases for $y_t > \theta$. Denote the wage share x_k as the k-th contract agreed upon at round kT evaluated at round $(k+1)T$. Then for all $x_k, x_{k-1} \in [0, \theta]$: $x_k = \pi^{-T}\left[ax_{k-1} + b\right]$, where $a = \frac{\delta}{1+\delta-\delta^T} < 1$ and $b = \frac{\left(1-\delta^T\pi^T\right)}{\delta^2\pi^2\left(1+\delta-\delta^T\right)}\theta$. Then $x^* = \frac{\left(1-\delta^T\pi^T\right)}{\delta^2\pi^2\left(\pi^T\left(1+\delta-\delta^T\right)-\delta\right)}\theta$ solves $\pi^T x^* = ax^* + b$ and $x^* \le \theta$ is equivalent to condition (4). Next, monotonic convergence to x^* on $[0, \theta]$ means that $x_k \leq x_{k-1}$ for all $x_{k-1} > x^*$ and $x_k \geq x_{k-1}$ for all $x_{k-1} < x^*$ for all k. Suppose $x_{k-1} > x^*$. Then $x_k \leq x_{k-1}$ follows from

$$\frac{x_k}{x_{k-1}} = \pi^{-T}a + \frac{\pi^{-T}b}{x_{k-1}} \leq \pi^{-T}a + \frac{\pi^{-T}b}{x^*} = \frac{1}{x^*}\left(\pi^{-T}ax^* + \pi^{-T}b\right) = \frac{x^*}{x^*} = 1$$

Similar arguments apply for $x_{k-1} < x^*$. So, there is monotonic convergence to x^* on $\left[0, \frac{1-\delta}{1-\delta\pi}\right]$. Finally, it is easy to verify that $x^* < \frac{1-\delta^T\pi^T}{\delta^2\pi^2\left(1-\delta^T\right)}\theta$. QED

Proof of Theorem 1. First, in the limit, condition (4) always holds, because

$$\lim_{\Delta\to 0}\delta^2\pi^2 \leq \lim_{\Delta\to 0}\delta^{-1}\left(\pi^{T+2}\left(\delta^2+\delta^3\right)+\delta^T\pi^T\left(1-\delta^2\pi^2\right)-1\right) \Leftrightarrow 1 \leq \left(2e^{\rho L}-1\right)$$

and the latter inequality holds iff $\rho L \geq 0$, which is assumed. Next, application of l'Hôpital's rule yields $\lim_{\Delta\to 0}\frac{1-\delta}{1-\delta\pi} = \frac{r}{r-\rho}$ and $\lim_{\Delta\to 0}\theta = \frac{r}{r-\rho}$. So, strike is credible for all $y_t \in \left[0, \frac{r}{r-\rho}\right]$. Then the limit expressions stated in the theorem follow trivially from Propositions 1, 2, 6 and 4. Furthermore, $\lim_{\Delta\to 0}x^* = \frac{1-e^{-(r-\rho)L}}{e^{\rho L}\left(2-e^{-rL}\right)-1}\cdot\frac{r}{r-\rho} < \frac{r}{r-\rho}$ iff $\rho L > 0$. The latter is assumed. Similarly, $\lim_{\Delta\to 0}x^* < \frac{1-e^{-(r-\rho)L}}{1-e^{-rL}}\cdot\frac{r}{r-\rho}$ iff $\rho L > 0$. Finally, $\left(\frac{1-e^{-(r-\rho)L}}{1-e^{-rL}}\cdot\frac{r}{r-\rho}, \frac{r}{r-\rho}\right] \neq \emptyset$ iff $\rho L > 0$. QED

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